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First Occurrence Prime Gaps

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Abstract. An ongoing search for first occurrence prime gaps continues.

An ongoing search for first occurrence prime gaps is being carried out which extends all previous work done on this subject. To date this search has found all such gaps for primes up to 7.263×10^{13}. First occurrence prime gaps had previously been known for primes less than 4.444×10^{12} [2]. Several gaps larger than the previously largest gap of 682 (not a first occurrence) found by Weintraub [4] have been found.

Computer programs were written in FORTRAN and CAL (Cray Assembly Language) on a CRAY-2 supercomputer.

The computation was conducted as follows. A sufficient number of primes were generated to perform a sieve. Odd numbers beginning with 3 were sieved one block at a time, where each block was chosen to contain 40,000,000 numbers based on system resource availability. The even numbers were eliminated during initialization of each block. One number was stored per 64-bit computer word. After sieving each block, the differences between consecutive primes were calculated and stored. This was accomplished by loading 64 elements of the sieved block at a time into a vector register. A 64-bit vector mask was created containing 1's for corresponding nonzero values in the vector register. If the vector mask was zero, the next 64 values of the sieved block were loaded into the vector register. If the mask was nonzero, an instruction to count the number of leading zeros was executed to get the offset from the beginning of the vector register for the next prime. A subtraction of the previous prime was done, thus arriving at the gap. The leftmost 1 of the vector mask was cleared and the method repeated, beginning with checking if the vector mask was zero. This was repeated until the entire block was processed. The last prime in each block was saved in order to calculate the difference between that prime and the first prime in the next block to make sure no gaps were missed. The time to perform the sieve for each block, and to calculate all gaps generated, was about 10.5 seconds for numbers in the range of 7.2×10^{13}. The largest prime in the last block processed to date is 72635119999997, so the table of first occurrence prime gaps is complete to that prime.

The following table lists all the first occurrence prime gaps found. This table agrees with all previously published results [2], [3], [1]. The maximal first occurrence prime gaps are marked with an asterisk (*).

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	gap	following the prime		gap	following the prime		gap	following the prime
*	2	3		102	1444309		202	107534587
*	4	7		104	1388483		204	112098817
*	6	23		106	1098847		206	232423823
*	8	89		108	2238823		208	192983851
	10	139		110	1468277	*	210	20831323
	12	199	*	112	370261		212	215949407
*	14	113	*	114	492113		214	253878403
	16	1831		116	5845193		216	202551667
*	18	523	*	118	1349533		218	327966101
*	20	887		120	1895359	*	220	47326693
*	22	1129		122	3117299	*	222	122164747
	24	1669		124	6752623		224	409866323
	26	2477		126	1671781		226	519653371
	28	2971		128	3851459		228	895858039
	30	4297		130	5518687		230	607010093
	32	5591	*	132	1357201		232	525436489
*	34	1327		134	6958667	*	234	189695659
*	36	9551		136	6371401		236	216668603
	38	30593		138	3826019		238	673919143
	40	19333		140	7621259		240	391995431
	42	16141		142	10343761		242	367876529
*	44	15683		144	11981443		244	693103639
	46	81463		146	6034247		246	555142061
	48	28229	*	148	2010733	*	248	191912783
	50	31907		150	13626257	*	250	387096133
*	52	19609		152	8421251		252	630045137
	54	35617	*	154	4652353		254	1202442089
	56	82073		156	17983717		256	1872851947
	58	44293		158	49269581		258	1316355323
	60	43331		160	33803689		260	944192807
	62	34061		162	39175217		262	1649328997
	64	89689		164	20285099		264	2357881993
	66	162143		166	83751121		266	1438779821
	68	134513		168	37305713		268	1579306789
	70	173359		170	27915737		270	1391048047
*	72	31397		172	38394127		272	1851255191
	74	404597		174	52721113		274	1282463269
	76	212701		176	38089277		276	649580171
	78	188029		178	39389989		278	4260928601
	80	542603	*	180	17051707		280	1855047163
	82	265621		182	36271601	*	282	436273009
	84	461717		184	79167733		284	1667186459
*	86	155921		186	147684137		286	2842739311
	88	544279		188	134065829	*	288	1294268491
	90	404851		190	142414669		290	1948819133
	92	927869		192	123454691	*	292	1453168141
	94	1100977		194	166726367		294	5692630189
*	96	360653		196	70396393		296	5260030511
	98	604073		198	46006769		298	8650524583
	100	396733		200	378043979		300	4758958741

	gap	following the prime		gap	following the prime		gap	following the prime
	302	6675573497		402	44293346177		502	1258535916601
	304	2433630109		404	144999022043		504	747431049203
	306	3917587237		406	49306638307		506	1339347750707
	308	5490459101		408	134664608389		508	1841086484491
	310	4024713661		410	98276144093		510	2209016910131
	312	6570018347		412	124221464119		512	1999066711391
	314	8948418749		414	49914935177	*	514	304599508537
	316	12109172293		416	121972158437	*	516	416608695821
	318	4372999721		418	129300694603		518	2296497058133
*	320	2300942549		420	82490815123		520	2336167262449
	322	7961074441		422	280974865361		522	1214820695701
	324	10958687879		424	264495345259		524	2256065636039
	326	5837935373		426	180265084403		526	1620505682371
	328	13086861181		428	219950168411		528	1529741785139
	330	6291356009		430	250964194171		530	2205492372371
	332	5893180121		432	87241770619	*	532	461690510011
	334	30827138509		434	127084569923	*	534	614487453523
*	336	3842610773		436	367459059871		536	5371284217763
	338	22076314313		438	101328529441		538	2122536905311
	340	8605261447		440	141846299801	*	540	738832927927
	342	12010745569		442	417470554687		542	2707053887651
	344	19724087267		444	36172730063		544	2652427555639
	346	11291401837		446	190418076203		546	2164206784721
	348	17002876643		448	402872474743		548	3380058341279
	350	16808773277		450	63816175447		550	2496646209271
	352	30750892801		452	466855187471		552	2210401546601
*	354	4302407359		454	202530831163		554	3621153039299
	356	24355072517	*	456	25056082087		556	4338624362173
	358	16792321339		458	304040251469		558	5263973982823
	360	20068818197		460	131956235563		560	4260199366373
	362	35877724601		462	400729567081		562	2081209441279
	364	25425617317	*	464	42652618343		564	1480064231153
	366	20108776097		466	565855695631		566	4897642179197
	368	51430518413	*	468	127976334671		568	6010330572331
	370	59942358571		470	681753256133		570	4442109925217
	372	20404137779		472	865244709607		572	5441175346967
	374	23064761663	*	474	182226896239		574	3108794067079
	376	16161669787		476	725978934347		576	8817792098461
	378	38116957819		478	367766547571		578	7552870120721
	380	23323808741		480	482423533897		580	9383081340541
*	382	10726904659		482	1051602787181	*	582	1346294310749
*	384	20678048297		484	767644374817		584	6993007248239
	386	35238645587	*	486	241160624143		586	6364466316577
	388	156798792223		488	1275363152099	*	588	1408695493609
	390	53241805651	*	490	297501075799		590	20761252261751
	392	117215204531		492	910361180689		592	3410069454097
*	394	22367084959		494	804541404419		594	5499789519863
	396	50806025873		496	880318998907		596	10487995154603
	398	40267027589		498	428315806823		598	5614481773561
	400	47203303159	*	500	303371455241		600	4872634110067

gap	following the prime	gap	following the prime	gap	following the prime
* 602	1968188556461	642	14141685364577	690	15712145060693
604	5439564948583	644	41433781612373	692	43603583701331
606	12112937821403	646	51027160468351	694	62088893223739
608	20767330530329	648	9787731507761	696	23333096984797
610	9105981382177	650	5120731250207	698	33785727371453
612	13397310636587	* 652	2614941710599	700	14998144209049
614	17418754709747	654	54916086007427	702	19786638118631
616	8095224517651	656	65862966031241	706	35625755878981
618	4165633395149	658	39883132551139	714	49639993268989
620	9344093035661	660	10653514291843	* 716	13829048559701
622	13059969946711	662	11082394066097	724	59692452738913
624	24923033918059	664	38745678640849	728	57723522921803
626	33605480400197	666	18691113008663	730	24179270588173
628	34140047613391	668	28340177964929	740	57360609786539
630	12644461143649	670	47137733785861	744	42610475373269
632	45678685880759	672	26456514142099	756	70099348325843
634	17659394869309	* 674	7177162611713	758	47581758352253
636	9483480841753	678	37970994487033	* 766	19581334192423
638	17499522060011	682	41459443375351	* 778	42842283925351
640	22099408494481	684	30236507704253		

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